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A DAMAGE-BASED DEFINITION OF EFFECTIVE PEAK ACCELERATION

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SUMMARY

This paper presents a rational basis for obtaining the Effective Peak Acceleration (EPA) of a given ground motion process. The proposed formulation considers the statistical variability in the ground motion, and is centred on the idea of explicitly linking EPA with expected cumulative damage in the structures due to the inelastic excursions. The structural behaviour has been modelled by a Single-Degree-of-Freedom (SDOF) bilinear hysteretic oscillator. EPA is considered to be the expected PGA of a scaled ground motion process such that this oscillator undergoes a specified expected damage under the unscaled process if it is linearly designed for the scaled process. For estimation of the damage, the oscillator has been replaced by an equivalent linear oscillator through stochastic averaging. A parametric study has been carried out to investigate the dependence of EPA on several governing parameters, and it has been shown that despite the strong dependence of EPA on oscillator time period, it may be possible to obtain 'period-averaged' EPA values for several ground motion processes for engineering applications. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: ground motion process; effective peak acceleration; peak ground acceleration; expected cumulative damage; bilinear hysteretic oscillator; stochastic averaging

1. INTRODUCTION

Peak Ground Acceleration (PGA) has been used over the years to provide a convenient anchor point for the design spectra specified by various regulatory agencies. However, it often does not correlate well with the damage caused in the engineered structures. The recorded accelerograms typically show one or two peaks of extremely high amplitudes which do not play significant role in the overall response of the structural systems (see References 1 and 2). The characteristic of the time history that leads to considerable damage and deformation in the structural systems is the repetitive motion with a strong energy content. The stray, high-frequency peaks of high amplitudes are only capable of causing localized damages. Thus, a better parameter representing the potential structural damage should depend on the total energy of the response process.[3,4] Moreover, obtaining the design spectrum by using PGA and an average spectral shape may introduce additional uncertainties (see Reference 5).

Despite the above-mentioned disadvantages, PGA-based ground motion characterization is still employed for the design purposes, perhaps due to overall convenience and better familiarity of the design engineers with PGA. To overcome the shortcomings of such a characterization, it has been a common practice to consider the 'Effective Peak Acceleration' (EPA) instead of PGA. The concept of EPA was first introduced by Newmark and Hall in some special studies and this may be defined as that acceleration which is most closely related to the structural response and to the damage potential of an earthquake (see Reference 6). No analytical

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formulation was proposed, and therefore, in the following years, the estimation of EPA corresponding to a given ground motion remained subject to the engineering judgement.

Despite being directly related to the structural damage, the EPA should also account for the stochastic nature of the ground motion. Due to the inherent uncertainties during the passage of seismic waves, there may be considerable statistical variations in the ground motions with the same energy distribution at a site. Gupta[7] accounted for such a stochasticity by using the order statistics of the acceleration peaks, and presented the first analytical formulation to estimate EPA. His formulation, however, did not consider the cumulative damage associated with the inelastic excursions of the yield level.

The purpose of this paper is to impart an analytical basis to EPA by explicitly relating the structural damage to EPA. For this purpose, EPA is defined to be the expected PGA of a scaled ground motion process such that a SDOF oscillator designed linearly for this process will suffer a specified expected damage on being subjected to the unscaled process. Further, the structural model has been assumed to be a bilinear, hysteretic, Single-Degree-Of-Freedom (SDOF) oscillator, and the stochastic formulation proposed by Basu and Gupta[8] has been used to estimate damage by considering its equivalent linear oscillator as proposed by Caughey.[9] Ground motion processes for nine different accelerograms have been considered and a parametric study has been carried out to study the effects of various parameters, e.g. initial time period, post-yield time period and damage, on EPA.

2. THE HYSTERETIC OSCILLATOR

Any estimation of the EPA for a given ground motion process requires an assessment of the structural damage caused by the inelastic excursions of the yield response level during this process. For this purpose, let us model the non-linear behaviour of the system by a SDOF, bilinear hysteretic oscillator. This oscillator is assumed to have an initial circular frequency of vibration, ω_0 and a post-yield frequency of vibration, $\alpha\omega_0$. It is also assumed to have a linear viscous damping coefficient, ζ. If the oscillator displaces by x relative to the ground under the base acceleration, $\ddot{u}_g$, its equation of motion can be written as follows:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + f(x,z) = -\ddot{u}_g \tag{1}$$

where, $\ddot{x}$ and $\dot{x}$, respectively, are the acceleration and velocity relative to the ground, and $f(x,z)$ denotes the acceleration term corresponding to the non-linear hysteresis. Modelling the hysteretic system as a combination of a linear and an elastoplastic system, $f(x,z)$ can be expressed as[10]

$$f(x,z) = \alpha\omega_0^2 x + (1-\alpha)\omega_0^2 z \tag{2}$$

where, z is obtained from the following differential equation:

$$\dot{z} = \dot{x}[1 - U(\dot{x})U(z - x_y) - U(-\dot{x})U(-z - x_y)] \tag{3}$$

Here, $U(.)$ denotes the Heaviside step function and x_y is the yield displacement of the hysteretic system. Further, α is the ratio of the post-yield to pre-yield stiffness, and is a measure of the stiffness degradation after the yielding occurs.

By statistical linearization, the above non-linear oscillator can be replaced by an equivalent linear oscillator with damping ratio, β_e and natural frequency, ω_e. These parameters can be evaluated by using the method of averaging proposed by Caughey.[9] This method is particularly useful for the lightly damped systems subjected to broad-band excitations. Thus, on representing the response with slowly varying amplitude and period, and on averaging over a cycle, the parameters of the equivalent system can be obtained as

$$\omega_e^2 = \omega_0^2 - \omega_0^2 g(\sigma)(1-\alpha) \tag{4}$$

and

$$\beta_{\mathrm{e}} = \beta\frac{\omega_0}{\omega_n} + \frac{1}{\sqrt{2\pi}}\frac{(1-\alpha)\omega_0^2}{\omega_{\mathrm{e}}^2\sigma}\left[1 - \mathrm{erf}\left(\frac{1}{\sqrt{2}\sigma}\right)\right] \tag{5}$$

where

$$g(\sigma) = \frac{1}{2\pi\sigma^4}\int_1^{\infty} A^3\left(\pi - \Lambda + \frac{1}{2}\sin 2\Lambda\right)\exp\left(-\frac{A}{2\sigma^2}\right)\,\mathrm{d}A \tag{6}$$

$$\cos\Lambda = 1 - \frac{2}{A} \tag{7}$$

erf(.) denotes the error function, and σ denotes the root-mean-square (r.m.s.) value of the response normalized with respect to the yield level.

The equivalent linear oscillator as above is a replacement of the original non-linear system in mean-square sense and thus, the square of the error between the responses of the two systems has been minimized. This approximation is reasonable for the response of a mildly non-linear oscillator. For such an oscillator, the response statistics of the equivalent linear oscillator are close to those of the original oscillator, and those may be easily estimated for a ground motion process characterized by its power spectral density function (PSDF) on using any formulation for the linear SDOF systems (e.g. see Reference 11).

3. EFFECTIVE PEAK ACCELERATION

It has been stated in the previous section how the response of a non-linear hysteretic oscillator can be obtained through the stochastic averaging technique. The damage in this oscillator may now be estimated by considering the formulation of Basu and Gupta.[8] This formulation is based on a simple elemental damage rule given by Miner,[12] and is summarized below for completeness in the paper.

Let the ith ordered peak as normalized with respect to the r.m.s. value, σ_x, of the response process be $X_{(i)}$. Let the yield response level, x_{y}, as normalized with respect to σ_x be denoted by l $(=1/\sigma)$. Using the conditional distribution of the higher-order peaks, the expected value of conditional damage due to the ith order of peak (excluding the largest order of peak) is obtained as

$$E(D_i|x_{(i-1)}) = \int_0^{x_{(i-1)\mathrm{eq}}} \frac{x_{(i)}^s}{Cl^s}(n-i+1)p(x_{(i)})\frac{[1-P(x_{(i)})]^{n-i}}{[1-P(x_{(i-1)\mathrm{eq}})]^{n-i+1}}\,\mathrm{d}x_{(i)} \tag{8}$$

where, the equivalent amplitude level for the $(i-1)$th peak is given by

$$x_{(i-1)\mathrm{eq}} = [E(D_{(i-1)}|x_{(i-2)})C]^{1/s}\,l \tag{9}$$

The expected damage due to the largest order of peak is obtained as

$$E(D_1) = \int_0^{\infty} \frac{x_{(1)}^s}{Cl^s}np(x_{(1)})[1-P(x_{(1)})]^{n-1}\,\mathrm{d}x_{(1)} \tag{10}$$

Here, s and C are the same positive empirical constants as in the failure law of the form

$$N\mu^s = C \tag{11}$$

where, N is the number of cycles at a constant ductility level, μ, to cause failure of a structural member. This law is in accordance with the popular Mason–Coffin relationship of plastic strain and low-cycle fatigue. Now,

if there are k number of inelastic excursions taking place beyond the yield level, l, the cumulative expected damage becomes

$$E(D_k) = \sum_{i=1}^{k} E(D_i | x_{(i-1)}) \tag{12}$$

The parameter, k, may be estimated by comparing the normalized yield level, l, with the normalized expected values of the higher-order peaks which are given by

$$E(X_{(i)}) = \int_{-\infty}^{\infty} \eta\, p_{(i)}(\eta)\, \mathrm{d}\eta \tag{13}$$

where, $p_{(i)}(\eta)$ is the probability density function of the ith-order peak. Using order statistics approach, this density function has been formulated by David,[13] Gupta and Trifunac[14] for the statistically independent peaks, and by Basu *et al.*[15,16] for the statistically dependent peaks. For simplicity in the proposed methodology, the results in this study are obtained by using the formulation of Gupta and Trifunac.[14] Further, the parameter, n has been estimated from the duration and moments of the power spectral density function (PSDF) of $x(t)$ as in Cartwright and Longuet-Higgins.[17]

Once the damage is estimated for the considered non-linear oscillator during a given ground motion process, it is necessary to estimate the largest scaling factor which may be applied to this process for the oscillator to respond linearly.

Let SD(ω_0, ζ) denote the expected spectral displacement of a linear oscillator with natural frequency, ω_0, and damping ratio, ζ, under the given ground motion process, $\ddot{u}_g$. Further, let the non-linear oscillator, having the initial natural frequency, ω_0, and damping ratio, ζ, be designed for a yield displacement, $l\sigma_x$. This oscillator will suffer a cumulative expected damage, $E(D_k)$, under the given process for k inelastic excursions. Now, let the given process be scaled such that the expected spectral displacement of the linear oscillator becomes equal to the yield displacement, $l\sigma_x$. The scaling factor to be applied is given by

$$a = \frac{l\sigma_x}{\mathrm{SD}(\omega_0, \zeta)} \tag{14}$$

The expected PGA corresponding to the scaled process, i.e. $a\ddot{u}_g$, is EPA by definition for an allowable value, say d, of the expected cumulative damage, $E(D_k)$. We may thus write

$$\mathrm{EPA} = \frac{\sigma_x}{\sigma \mathrm{SD}(\omega_0, \zeta)} \mathrm{PGA} \tag{15}$$

It may be noted that the non-linear oscillator is expected to respond elastically to the process, $a\ddot{u}_g$, while it will suffer an expected cumulative damage, $E(D_k)$, under the given process, $\ddot{u}_g$. Equation (15) may be used to estimate the EPA for a given ground motion process, once ω_0, ζ, α, and d are specified. The proposed formulation can be easily extended to other damage models and non-linear oscillators.

4. NUMERICAL RESULTS

The above formulation has been used to calculate EPA for several oscillators of different characteristics. A variety of ground motion processes corresponding to different accelerograms have been considered for this purpose. The parameters in equation (11) have been assumed as $C = 416$ and $s = 6$ to represent concrete structures (as reported by Yamada[18]). The linear viscous damping in all the cases has been assumed to be 5 per cent of the critical. Further, for the characterization of a given ground motion process through PSDF, elastic response spectrum has been calculated for the corresponding accelerogram. Assuming this to be the expected response spectrum, the iterative scheme of Kaul[19] and Unruh and Kana[20] has been employed to

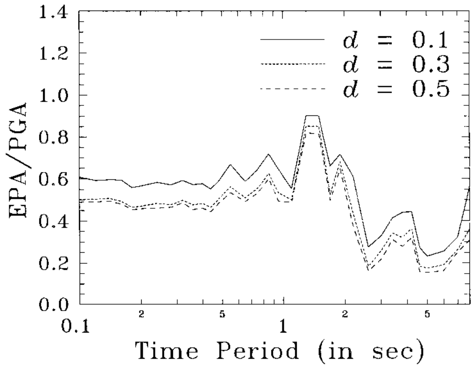

Figure 1. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}1$ for Michoacan earthquake

obtain the response spectrum-compatible PSDF. Such a PSDF accounts for the non-stationarity in the input ground motion as well as the transient nature of the response in a convenient manner.

The results have been obtained in form of a set of curves showing the variation of EPA/PGA (see equation (15)) with the initial natural period ($=2\pi/\omega_0$) of the non-linear oscillator. In each set, there are three uniform damage curves corresponding to damage, $d = 0{\cdot}1, 0{\cdot}2$ and $0{\cdot}3$. Figures 1 and 2 show the sets of these curves for oscillators with $\alpha = 0{\cdot}1$ and $0{\cdot}5$, respectively, representing the oscillators with nearly elastoplastic and moderately hysteretic characteristics, for the synthetically generated accelerogram at the Mexico City site corresponding to the 1985 Michoacan earthquake (see Reference 21 for details). Similar curves are plotted in Figures 3 and 4 for the recorded N85E component at the Cholame site for the 1966 Parkfield earthquake, and in the Figures 5 and 6 for the recorded S00E component at the El Centro site for the 1940 Imperial Valley earthquake. All the curves in these figures show that the variation of the ratio, EPA/PGA with the period is arbitrary for a particular ground motion process and thus, no trend is obvious between the curves for different processes. The curves in a figure though depend on the characteristics of the ground motion, there is no apparent correlation between the energy distribution of the ground motion and the fluctuations in the EPA/PGA with the time period. The lower values in these figures, however, indicate that at the corresponding periods, the ground motions have been much less damaging to the oscillators and therefore, smaller scaling factors may be allowed in the PGAs for the uniform damage. It may, however, be observed that for a given ground motion process, these fluctuations of the curves tend to smooth out with the increase in α. Thus, for the oscillators with small to moderate hysteresis, the ratio, EPA/PGA, becomes relatively constant throughout the period range under consideration. The values of EPA/PGA greater than unity at some periods, e.g. see Figures 3 and 4, indicate that the oscillator designed to respond linearly at the expected PGA level may suffer some damage due to statistical variations in the PGA value. It is also interesting to note that with the increase in the damage level from 0·3 to 0·5, there is little change in the values of EPA while this change is relatively more significant for the increase in the damage level from 0·1 to 0·3. To study this observation in greater detail, the 1 s initial period oscillator is considered and the variation of EPA/PGA with the damage level is shown in Figure 7 for $\alpha = 0{\cdot}1$ and $0{\cdot}5$ in case of the El Centro ground motion. It is seen that beyond the damage level around 0·3, an increase in the ratio, EPA/PGA, is associated with a sudden increase in damage for both curves. Similar observations are also obtained for the Parkfield and the Michoacan ground motions. Thus, this damage level around 0·3 appears to be a 'critical damage level', and the EPA corresponding to this becomes the 'critical EPA' such that any further reduction in EPA below this may lead to a significant increase in the damage of structures. The curves in Figure 7 also show that a greater value of α may lead to greater damage. However, for low values of allowed damage, this increase may be insignificant.

To further investigate the effect of α on EPA/PGA, curves representing the variation of EPA/PGA with α are obtained for $d = 0{\cdot}1, 0{\cdot}2$ and $0{\cdot}3$ in case of the El Centro ground motion (see Figure 8). The oscillator is assumed to have an initial period of 0·5 s. It is seen that the slopes of all the curves are reasonably mild, thus

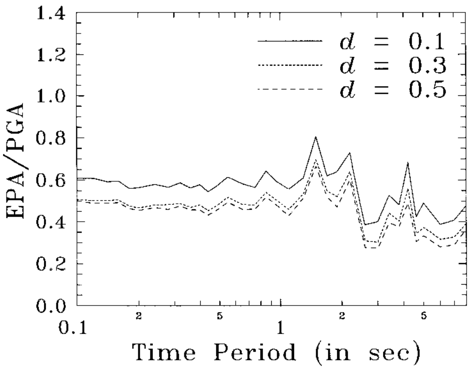

Figure 2. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}5$ for Michoacan earthquake

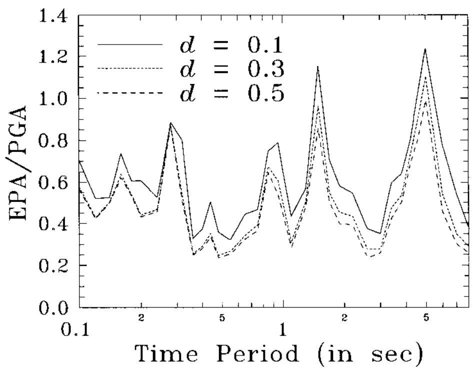

Figure 3. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}1$ for Parkfield earthquake

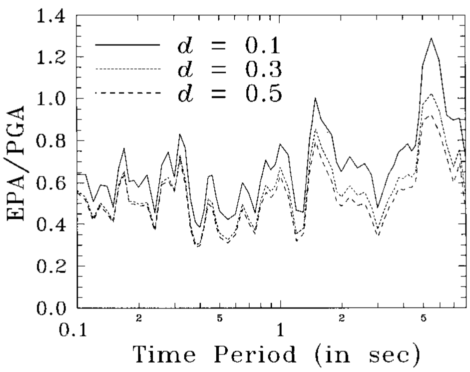

Figure 4. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}5$ for Parkfield earthquake

indicating marginal changes in the EPA/PGA ratio with a large increase in α. Thus, in this range of damage, the ratio, EPA/PGA can be assumed to be almost invariant of α.

The above observations clearly indicate that EPA is not invariant of the initial time period of the oscillator. However, it is an accepted practice to specify constant EPA value for a ground motion since the period-independent EPA values lead to considerable simplicity in application and are thus popular with the practising engineers. Keeping this in view, averaged values of EPA/PGA over the period range 0·1–8 sec have been tabulated in Tables I and II for $\alpha = 0{\cdot}1$ and 0·5, respectively. In each table, these averaged values are shown

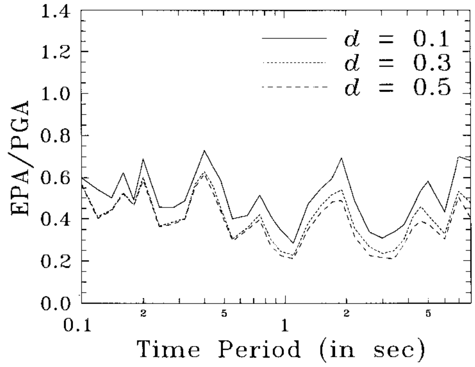

Figure 5. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}1$ for Imperial Valley earthquake

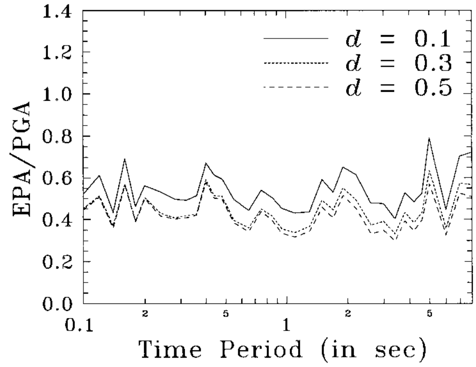

Figure 6. EPA/PGA vs. initial time period with $\alpha = 0{\cdot}5$ for Imperial Valley earthquake

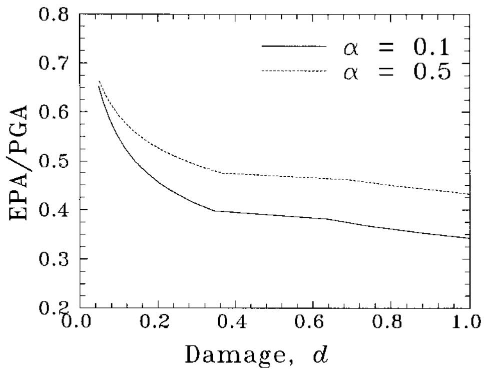

Figure 7. Variation of EPA/PGA with d for 1 s time period oscillator

for $d = 0{\cdot}1, 0{\cdot}3$ and $0{\cdot}5$ in case of six different recorded motions in addition to the three records considered above. Besides the 'mean' values, the Coefficient of Variation (COV) values are also tabulated to reflect the relative variability in the values of EPA. It is observed that the coefficient of variation is 0·2–0·3 in most of the cases which may be acceptable for most of the engineering applications. However, this coefficient may become as high as 0·5 in case of San Fernando ground motion. In such cases, averaging may have to be done over a narrower range of time periods in which the periods of structural systems of interest are likely to lie.

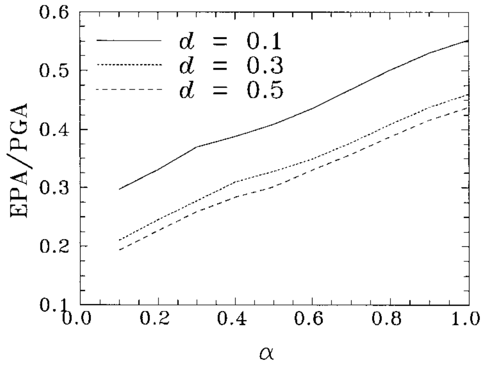

Figure 8. Variation of EPA/PGA with α for 0·5 s time period oscillator

Table I. 'Averaged' EPA/PGA ratios for hysteretic SDOF oscillators with $\alpha = 0{\cdot}1$

Ground motion record	$d = 0{\cdot}1$		$d = 0{\cdot}3$		$d = 0{\cdot}5$	
	Mean	COV	Mean	COV	Mean	COV
1985 Michoacan earthquake Mexico City site synthetic	0·56	0·30	0·47	0·34	0·45	0·36
1966 Parkfield earthquake Cholame site N85E component	0·50	0·31	0·41	0·35	0·38	0·37
1940 Imperial valley earthquake El Centro site S00E component	0·51	0·25	0·42	0·28	0·39	0·29
1952 Kern county earthquake Taft site N21E component	0·60	0·33	0·50	0·35	0·46	0·34
1971 San Fernando earthquake Pacoima dam site S16E component	0·62	0·45	0·51	0·50	0·47	0·50
1987 Whittier Narrows earthquake Santa Fe springs site S60E component	0·49	0·31	0·40	0·34	0·37	0·36
1979 Coyote lake earthquake Gilroy site S40E component	0·47	0·17	0·38	0·21	0·35	0·24
1980 Mammoth lakes aftershock Mammoth lakes site S34E component	0·48	0·37	0·40	0·43	0·37	0·46
1984 Morgan hill earthquake Coyote lake dam site N75W component	0·45	0·23	0·36	0·27	0·34	0·30

In the above discussion, it has been implicitly assumed that the value of parameter, d is known a priori to the designers. However, since the values of d as considered here represent low or high damages in qualitative sense only, it is necessary to correlate the observed structural damages during the past earthquakes with the EPA values obtained based on engineering judgement. Once a value of d is fixed depending on the amount of expected damage we may want to allow in a particular class of structures, the proposed formulation and

Table II. 'Averaged' EPA/PGA ratios for hysteretic SDOF oscillators with $\alpha = 0{\cdot}5$

Ground motion record	$d = 0{\cdot}1$		$d = 0{\cdot}3$		$d = 0{\cdot}5$	
	Mean	COV	Mean	COV	Mean	COV
1985 Michoacan earthquake Mexico city site synthetic	0·58	0·16	0·48	0·19	0·46	0·20
1966 Parkfield earthquake Cholame site N85E component	0·55	0·20	0·45	0·22	0·43	0·22
1940 Imperial valley earthquake El Centro site S00E component	0·55	0·18	0·45	0·20	0·43	0·20
1952 Kern county earthquake Taft site N21E component	0·64	0·26	0·53	0·28	0·50	0·27
1971 San Fernando earthquake Pacoima dam site S16E component	0·67	0·42	0·56	0·48	0·52	0·42
1987 Whittier Narrows earthquake Santa Fe springs site S60E component	0·53	0·22	0·44	0·25	0·42	0·26
1979 Coyote lake earthquake Gilroy site S40E component	0·54	0·11	0·45	0·12	0·42	0·12
1980 Mammoth lakes aftershock Mammoth lakes site S34E component	0·53	0·21	0·44	0·24	0·41	0·25
1984 Morgan hill earthquake Coyote lake dam site N75W component	0·49	0·29	0·42	0·15	0·39	0·16

averaging of EPA/PGA ratios over a period range would ensure consistency in the fixing of the EPA value for any new ground motion process.

5. CONCLUSIONS

A stochastic formulation has been proposed for obtaining the EPA corresponding to a specified damage level in case of a given ground motion process. Numerical results have clearly shown that the EPA so calculated decreases with the increase in the allowable damage. However, to avoid disproportionately high damages, EPA should be restricted to a critical EPA. Further, it has been shown that this EPA also fluctuates with the initial time-period of the oscillator in an irregular manner, and is found to be more for less hysteretic structural systems. However, the dependence of EPA on time period of the oscillator can be neglected for several earthquake processes, especially in case of the moderate to less hysteretic systems with lower allowable damage levels. Hence, using the 'period-averaged' EPA values appears to be appropriate for several engineering applications. The period range, over which this averaging is done, is usually large, but this may sometimes be quite narrow, depending on the ground motion process and on the range of structural periods of interest.

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